AC 2007-858: AN INTRODUCTORY ENGINEERING DESIGN PROJECT UTILIZING FINITE ELEMENT ANALYSIS AND RAPID PROTOTYPING

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An Introductory Engineering Design Project
Utilizing Finite Element Analysis and Rapid Prototyping

Abstract

Design projects are often used to stimulate interest in engineering among high school students (and to sustain interest among engineering freshmen). A typical model for these projects is “design-build-test,” in which students evaluate their designs by making and testing a physical prototype. While these projects can be valuable in motivating many students, a danger is that they ignore any analysis steps, giving the impression that engineering design is a strictly trial-and-error process. This impression is contrary to modern engineering design practice, in which modern analysis tools are used in conjunction with rapid prototyping methods to allow for fast design iterations. In this paper, a “design-analyze-build-test” project is described. Easy-to-use solid modeling and finite element analysis software was used along with a low-cost rapid prototyping system in a project in which high school students attempted to optimize the design of a component subjected to well-defined loading and constraints.

This project was conducted as part of Summer Ventures, a program in which talented high school students from across North Carolina explore math and science-related careers at several University of North Carolina System campuses. This was the initial offering of engineering as an option for the students. Students selected three areas of participation. For three weeks, they spent two hours per day in each of the selected areas. During the fourth and final week, they chose one of their three areas for more in-depth study.

During the engineering portion of the first three weeks of the program, lessons included both mechanical and electrical concepts. In the mechanical portion, students learned how to use solid modeling software and were introduced to finite element analysis. To gain an understanding of the finite element method, the students hand-worked problems consisting of assemblies of springs. Given the element matrices of the springs, students assembled the system stiffness matrix, applied boundary conditions, and solved for the unknown displacements. Results compared very closely to those of a physical assembly of high-precision extension springs. The students then modeled a baseline design of a bracket, following step-by-step instructions, and analyzed the performance of the bracket under a weight load using finite element analysis software. Modulus of elasticity and strength values were determined from tensile and compressive tests conducted on the bracket material. The baseline bracket model was built on a rapid prototyping machine and tested by hanging weights on it until it failed. Students then were asked to improve on the baseline design, with the goal of producing the lightest-weight design that achieves a given factor of safety under the design load and meets a specified deflection limit. The students’ designs were then built and tested.

In addition to introducing students to the quantitative side of the engineering design process, this project also emphasized the role of uncertainty in the process. Students were asked what factor of safety they felt comfortable in specifying, and a discussion on the trade-offs between risk and cost and the role of design codes followed.
Summer Ventures Program

Summer Ventures in Science and Mathematics is a state-wide program of the University of North Carolina System. Rising high school juniors and seniors are eligible to apply for the program, and participants are selected based on academic ability, motivation to study math and science topics, and emotional maturity. Students spend four weeks in residence at one of six participating UNC campuses. During the first three weeks, students work in three subject areas, chosen from a list of 16 program offerings, such as Archaeology, Biology Topics, Chemistry Topics, Mathematical Modeling, Environmental Sciences, Computer Science, Medicine, and Marine Sciences. Students indicate their top four choices on their application, and the selection committee attempts to place students at the campus offering their top choice whenever possible. During the fourth week, students choose an area for independent study. A research paper and presentation are required for all students. Room, board, and tuition costs are completely paid by the state. Participating students are responsible only for costs of transportation to and from the campus and incidental expenses.

The Summer Ventures program has been offered since 1984. In that time, the program has become very popular, and students selected for the program are high academic achievers. Engineering was added as a program choice for the first time in 2006. Seventeen students chose to participate in the engineering area.

Engineering Design Content

In designing content for this program, we examined other engineering-related summer outreach activities. The ASEE Engineering K12 Center lists approximately 70 universities and organizations that conduct engineering outreach activities to K-12 students. The Engineering Education Service Center maintains a directory of pre-engineering summer camps, with over 150 entries. Many of these programs involve some type of design/build/test project, such as toothpick-bridge-building, mousetrap-car, or egg-drop contests. One drawback of this type of project is a lack of connection to the real world of engineering design including use of the modern tools of engineering. Solid modeling and rapid prototyping have been used in a summer activity at Milwaukee School of Engineering, but the design is for aesthetics only. Programs at the University of Wisconsin-Madison and University of Virginia have successfully integrated mathematics and science into design activities. The length of the Summer Ventures Program and the level of academic achievement of the students encouraged us to add additional academic rigor to the hands-on activities of a typical summer camp.

The engineering program was divided into six modules:

1. Solid Modeling Introduction
2. Design, Analysis, and Testing of Bracket
3. PLCs and Electro-Pneumatics
4. Digital Logic and PID Control
5. Motion Analysis of 4-Bar Linkage
6. Rapid Prototyping and Free Design

This paper will focus on the activities of the second module.
Design, Analysis, and Testing Module

In the solid modeling module, students worked through several tutorials, learning the basics of part design and assembly, using SolidWorks® software. The first activity of the Design, Analysis, and Testing module was the creation of the bracket baseline design, which is shown in Figure 1.

Before analyzing the bracket with FEA, the students were introduced to the finite element method through a problem involving an assembly of four linear springs, as illustrated in Figure 2. Spring constants for the problem are: \( k_1 = 30 \) lb/inch, \( k_2 = 10 \) lb/inch, \( k_3 = 1000 \) lb/inch, and \( k_4 = 40 \) lb/inch.

The steps of the solution to this problem and its solution were presented in a series of steps as follows:

1. Explain the concepts of a linear spring, force, deflection, and the spring constant.
2. Develop the rules for replacing springs in series or in parallel with an equivalent spring.
3. Solve for the deflection of the assembly (node 5) using equivalent spring rules. Point out that the very stiff spring (spring 3) has a minor impact on the system spring stiffness, as it behaves as a nearly rigid member.
4. Introduce the concept of the stiffness matrix of an element (spring), relating the displacements of the ends to the forces applied at the ends.
5. Starting with the concept of equilibrium at a node, develop the method for assembling the system stiffness matrix.
6. Introduce the concept of boundary conditions, with the important idea that for every degree of freedom, either the displacement or the load, but not both, must be defined. Students can usually identify that the displacements of nodes 1 and 2 and the load at node 5 are known, but often do not immediately grasp that the external loads at nodes 3 and 4 are known (value of zero).
7. Apply the boundary conditions, so that the equations are reduced to a series of three linear equations, with the known forces on one side of the equations and the unknown displacements on the other side.
8. Solve the equations for the unknown displacements. The matrix algebra is performed with a spreadsheet. The displacement of node 5 is compared to the value calculated in step 3. Point out that \( D_3 \) and \( D_4 \) are nearly equal, as the stiff spring 3 stretches much less than the others.
9. Substitute the five known displacements into the system equations of step 5, and calculate the forces at the nodes. Forces $F_1$ and $F_2$ are the reaction forces, and balance the 20-pound applied load. The other forces equal the values of the applied nodal forces, and represent a check of the displacement solution.

10. Apply the known displacements to each of the element equations to find the force in each spring. Point out that the entire 20 pounds must be resisted by both spring 3 and spring 4 (in series), while the load is distributed between springs 1 and 2 proportional to their stiffness (spring constant) values.

After working through the previous example, students were asked to solve for the nodal displacements and forces of the assembly shown in Figure 3. In this case, the spring constants were for a set of precision tensile springs that were purchased from an industrial supply distributor.

![Figure 3 Spring Assembly Model for Student Solution](image)

Students then compared their solution to a physical assembly of the springs, as shown in Figure 4. A 5-N preload was applied to the system to remove any slack from the system, and then the load was increased to 15 N. The difference in the position of the wooden block (Node 3) was noted and compared to the calculated displacement due to a 10-N load. Since the model of this system is one-dimensional (displacements and forces in the x-direction only), the scale is attached to the wooden block at the position which eliminates rigid-body rotation.

![Figure 4 Spring Assembly Test](image)
Results from this test were surprisingly accurate, considering the simplicity of the setup. Deflection values agreed with the calculated values to within 1 or 2 mm. It should be noted that the students did not find this exercise to be completely straight-forward. Because of the node numbering scheme, several made mistakes in assembling the system stiffness matrix or applying the boundary conditions. However, these errors provided an opportunity for teaching the importance of checking the reasonableness of the solution. One common error involved placing the terms of spring 4’s stiffness matrix in the wrong locations in the system matrix (as if spring 4 connected nodes 3 and 4 rather than nodes 4 and 5). In this case, the calculated displacement for node 5 was zero, which can be observed to be incorrect. Another error was to eliminate the first and second rows and columns from the reduced stiffness matrix, rather than the first and fourth rows and columns (corresponding to zero deflections at nodes 1 and 4). In this case, the deflections of all three non-constrained nodes were predicted to be the same, indicating rigid body translations of springs 3 and 4. Once the errors were corrected, the students were pleased to see that their models accurately predicted the response of the system.

Before proceeding to the analysis of the bracket, students were introduced to the concepts of stress and strain, and witnessed a tensile test of a bar fabricated on a rapid prototyping machine. The machine used was a Dimension SST fused deposition modeler.\(^8\) This machine produces parts from ABS plastic. Water-soluble support structures allow parts to be produced with almost no post-processing. A stress-strain graph from the tensile test is shown as Figure 5. From the graph, a yield strength of about 2800 psi was determined. A curve-fit of the linear portion of the curve resulted in a modulus of elasticity of about 240,000 psi. These were the properties used in

![Stress-Strain Curve from Tensile Test of Rapid Prototype Material](image)
the FE analysis, along with an assumed Poisson’s ratio of 0.4 (a typical value for plastic).

The students then stepped through an analysis of the bracket using the COSMOSWorks® FEA program, which is integrated into SolidWorks. The analysis was to simulate the test setup shown in Figure 6. The bracket was attached to a wooden 2X4 which was held to a table with clamps. Weight was hung from the screw eye, and load transferred to the bracket through a washer.

In setting up the analysis, the fact that assumptions had to be made was emphasized. In this case, the assumption was made that the 2X4 would be held rigid, and the screws would hold the back face of the bracket flush against the 2X4. Accordingly, a fixed boundary condition was applied to the FE model. Also, it was assumed that the load would be distributed evenly over the surface of the bracket under the washer. Therefore, a uniform pressure was applied to that region, so that the total downward force equaled five pounds. Students found the predicted deflection of the end of the bracket, and examined the stress distribution.

When examining the Von-Mises equivalent stresses in the bracket, it was seen that the highest stress area was near the top of the hole passing through the ribs, as shown in Figure 7. The baseline bracket was designed so that design improvements would be easy to make. The thin area above the hole was clearly the critical region. The ribs were placed above the loading point rather than below so that the highest stress would be tensile and therefore more likely to cause catastrophic failure.

Before testing the rapid-prototyped bracket, the failure load was predicted from the analysis results. If the Von-Mises results were used, then a factor of safety of 3.98 was computed for the five-pound load, and so a failure load of 19.9 pounds was predicted. However, it was pointed out to the students that the Von-Mises criterion is used to predict yielding of ductile materials. The tensile test of the RP material appears to show some yielding, but the load drops off quickly after reaching its peak value. In other words, the material behavior does not neatly fit either an idealized brittle (straight line to failure) or ductile perfectly elastic-plastic (straight line to yield, then flat line until ultimate failure) model. A compressive test of
the material showed a strength of about 4,500 psi. Based on the different values of tensile and compressive stresses, as well as the relatively low strain to failure of the tensile test, it was decided to use a brittle failure criterion, the Coulomb-Mohr criterion. This resulted in a factor of safety of 3.2, and therefore a predicted failure load of 16.0 pounds.

The bracket load test was conducted by hanging a bucket filled with water from the end of the bracket. Initially, the weight of the bucket and water was five pounds. A dial indicator was used to measure the deflection under load, as shown in Figure 8. Water was then slowly poured into the bucket until failure occurred and the bucket and water were then weighed. We noted that the use of the water as the load gave students a good feel for the force resisted by the bracket, as opposed to seeing the read-out from a digital load cell. For the baseline design, the failure load was 13 pounds, less than the value predicted by the FE analysis but within a reasonable order of magnitude. The deflection measured was about 25% greater than predicted.

An examination of the failed bracket revealed that the failure surfaces were smooth, as compared to the rough failure surfaces of the tensile test specimens. This led to the hypothesis that the strength of the RP part was not uniform in all directions. In particular, the bracket was built in the orientation shown in Figure 9, to minimize the amount of support material needed. The material is put down in layers parallel to the xy-plane, and the failure of the bracket occurred between the layers. However, the tensile specimen was built laying flat, parallel to the build plane. Therefore, the strength of the material between layers (in the z-direction) appeared to be lower than the strength within layers (in the xy-plane). This hypothesis was later confirmed by testing tensile specimens built in different orientations. While this strength difference could be compensated for somewhat by building the brackets in a different orientation, or could be accounted for by utilizing a 3-D failure criterion similar to that used for composite material analysis, neither of these options was desirable. The “upright” build orientation allowed for several brackets to be built simultaneously, and was the most practical way to build the student designs within the needed time frame. Introducing a failure criterion beyond those available within the FE program would be
possible, but probably beyond the students’ level of understanding. Therefore, the strength differences were treated as another imperfection in the idealized model.

Students then worked in groups of two to redesign the bracket. Design constraints included:

- No modifications to the back plate of the bracket
- Overall size limitations
- Maintenance of the open space through the bracket to allow for wiring to pass through

The goal was to meet deflection and factor of safety requirements while minimizing the weight of the bracket.

Figure 10 shows two students with their FEA results and their rapid prototyped bracket. Overall, the students were able to predict the location of the failures accurately, although failure loads often varied from the predicted loads by as much as 50%.

When the results of the project were analyzed, emphasis was placed on the role of engineering judgment in the design and analysis process. Even with the modern tools that are available to solve problems numerically, it is the definition of the problem and interpretation of the results that define the role of an engineer. Rapid prototyping and finite element analysis are technologies that serve two seemingly opposite approaches to engineering design. Rapid prototyping is often characterized as a tool that allows physical testing to be performed early and often in the design process, allowing for multiple concepts to be evaluated and mistakes to be avoided. Finite element analysis is often advertised as a way to avoid physical testing, relying instead on “virtual testing.” These two approaches can of course be combined into a process that uses FEA to reduce the number of tests required while using RP to verify the analysis results and discover unanticipated factors (such as the discovery of the unequal strengths in this project).

![Figure 10 Student-Designed Bracket](image-url)
Assessment

At the end of the program, students were asked to complete a short survey, rating each of the components of the engineering activities by how much they learned and how interesting and fun the activities were. Student responses were on a five-point scale: 5 = Strongly Agree, 4 = Agree, 3 = Neutral, 2 = Disagree, and 1 = Strongly Disagree. All 17 students completed a survey form.

The Design, Analysis, and Testing module received the highest ratings for learning something new, as shown in Table 1. Sixteen of the 17 students selected “Strongly Agree” or “Agree” for this statement.

<table>
<thead>
<tr>
<th>Module</th>
<th>Average Rating</th>
<th>% of 4 or 5 Ratings</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Design, Analysis, and Testing of Bracket</td>
<td>4.71</td>
<td>94%</td>
</tr>
<tr>
<td>5. Motion Analysis of 4-Bar Linkage</td>
<td>4.59</td>
<td>94%</td>
</tr>
<tr>
<td>6. Rapid Prototyping and Free Design</td>
<td>4.59</td>
<td>88%</td>
</tr>
<tr>
<td>1. Solid Modeling Introduction</td>
<td>4.47</td>
<td>88%</td>
</tr>
<tr>
<td>3. PLCs and Electro-Pneumatics</td>
<td>4.29</td>
<td>82%</td>
</tr>
<tr>
<td>4. Digital Logic and PID Control</td>
<td>4.29</td>
<td>82%</td>
</tr>
</tbody>
</table>

The Design, Analysis, and Testing module also received high ratings for being fun/interesting, as shown in Table 2. The only module rated higher was the Rapid Prototyping and Free Design module, in which students were allowed to design anything they wanted (within size limits) and have their design prototyped. The students were able to keep their prototyped designs as souvenirs.

<table>
<thead>
<tr>
<th>Module</th>
<th>Average Rating</th>
<th>% of 4 or 5 Ratings</th>
</tr>
</thead>
<tbody>
<tr>
<td>6. Rapid Prototyping and Free Design</td>
<td>4.88</td>
<td>100%</td>
</tr>
<tr>
<td>2. Design, Analysis, and Testing of Bracket</td>
<td>4.29</td>
<td>82%</td>
</tr>
<tr>
<td>1. Solid Modeling Introduction</td>
<td>4.12</td>
<td>88%</td>
</tr>
<tr>
<td>5. Motion Analysis of 4-Bar Linkage</td>
<td>4.06</td>
<td>82%</td>
</tr>
<tr>
<td>3. PLCs and Electro-Pneumatics</td>
<td>3.71</td>
<td>59%</td>
</tr>
<tr>
<td>4. Digital Logic and PID Control</td>
<td>3.47</td>
<td>53%</td>
</tr>
</tbody>
</table>

Six of the students chose to do engineering-related projects during their independent-study week. One performed a systematic study to optimize the design parameters of the bracket. Another studied the strength values of the RP material by building and testing tensile bars oriented in different configurations on the platform. His results confirmed that the strength and stiffness in the x- and y-directions were approximately equal, while the strength and stiffness in the z-direction was lower.
Conclusions

This project was successful in teaching talented high school students about engineering design, using modern software along with rapid prototyping. The students responded well to a project that correlated mathematics and science to the hands-on activities that are typical of many summer programs.

References

7. SolidWorks is a registered trademark of the SolidWorks Corporation, 300 Baker Avenue, Concord, MA 01742.
8. Dimension SST is a product of The Dimension 3D Printing Group, a business unit of Stratasys, Inc., 14950 Martin Drive, Eden Prairie, MN 55344.
9. COSMOSWorks is a registered trademark of the SolidWorks Corporation, 300 Baker Avenue, Concord, MA 01742.