Revising the Flipped Classroom

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An Alternative to the Flipped Classroom

Abstract
A novel approach to teaching core Mechanical Engineering courses is presented. The approach combines the Flipped Classroom method with Visual Thinking Strategy. It exploits the powerful aspects of Flipped Classroom, including interactive and engaged problem-solving sessions, while avoiding its pitfalls, in particular the main students’ complaint which is the elimination of interaction with the instructor while encountering the material for the first time. This combined approach was developed using step-by-step feedback and active guidance from Mechanical Engineering students and was devised to fit their needs and expectations in learning, retaining, and applying the material. The technique was tested using two parallel sections of Mechanics of Materials, one using the proposed technique and the other the traditional lecture method as control section. The effectiveness of the two teaching approaches were compared using common exam problems evaluated with the same rubric. The paper provides the details of the proposed technique, supporting background in neuroscience of learning, sample pre-class exercises, and a narrative of constructive classroom conversation from one of the sessions. The result showed enhancement of student engagement and participation, facilitation of learning, and favorable student feedback.

Introduction
A brief history of Flipped Classroom. Flipped Classroom, is a teaching methodology in which instead of learning the material in class and doing homework at home, the students watch premade videos of the topics at home and spend the class time on working problems. The
concept of a flipped classroom was inspired by teachers Jonathan Bergman and Aaron Sams. They developed “reversed instruction” by offering PowerPoint presentations online for students who had missed class [1].

Since then, the concept has been developed into a teaching methodology, interpreted and applied in various ways. Variations take two separate routes. One category of variations focusses on the means of delivering premade lectures that students watch at home, using different online platforms and technology. Developing and delivering premade lectures is well explored. For a list of tools and technology see [2]. The second category of variations addresses the issues and shortcomings of flipped methodology. This area is less explored and the tried schemes so far offer limited remedies for the shortcomings. In this section, we will discuss these issues and tried methods, and then propose a solution based on existing brain research on learning.

**Shortcomings of flipped methodology.** Widespread student feedback, across many higher-education institutions, suggests that while an overwhelming majority of students find the in-class active problem-solving effective and enjoyable, the majority of students actively dislike the fact that with a flipped classroom the learning must be initiated and accomplished at home, without instructor support.

This issue is fundamental and its roots can be understood by a study of the existing research on the neuroscience of learning, but before that overview, it is important to mention that by addressing the out-of-class part of the flip, we are not undermining the great value of online learning. Many learners have experienced deep and lasting learning through the myriad of courses available online, especially, when we opt for online learning in the areas of continuing education and professional development, when the needed networks already exist in the learners’ brain, and the motivation is already at work. The focus of this article is on offering of a method to make the flipped classroom more effective and more desired by the students.
**Relation to neurobiology of learning.** Let’s start with the work of Professor James E. Zull on neurobiology of learning [3]. Professor Zull, when in charge of enhancing teaching in Case Western University, observed a diverse group of Case Western instructors, their methods of teaching, and the response of their students to them. He then applied his background knowledge in brain neurobiology to develop a theory with practical teaching implications for enhancement of student learning. Two practical aspects of Zull’s research shed light on and lead to mitigating strategies for the difficulties with the flip. The first aspect relates to how learning happens in the brain, by creation of new brain networks and/or by extending and linking the existing ones. Both the creation of new and extension of existing networks can be enhanced by an instructor’s effective usage of the knowledge of the working of the brain. In particular, if a new topic is to be learned by creation of new networks, experiential learning, using as much sensory data as possible, is the most effective path [4]. This means stimulating the sense of touch, as in bringing a model to class and letting the student work with it, as well as auditory and visual, plus interaction with the instructor and fellow students, channels that are missing in the out-of-class part of the flip.

On the other hand, another effective path of learning can be the evoking of existing networks in the students’ brains, and then building upon them. This facet, in particular, needs active interaction in class and usually starts with the instructor evoking a close-to-life example, something that students have previous experience with. As an example, once in a dynamics class, we started the teaching of mechanical impact with a personal true story that put a comical spin on a car collision experience. This story evoked existing networks in the audience as each student referred to their own memories. Brain networks that are “turned on” at the same time, connect, a fact which in this case allows the learning of mechanical impact to build on a strong existing network, making the retention and recall of the material much easier. This technique, however, relies on the ability of the instructors to evoke existing networks in all of their students. The individuals sitting in the classroom with different backgrounds and different memories won’t necessarily connect to the same story the same way. Some students may not connect with a particular story at all and this lack of an experiential connection is one of the reasons that this process cannot be delegated to premade videos. Whether or not a student connects with a part of
the lesson may seem hard to judge in terms of criteria, but the attentive instructor is bound to notice the visual cues, and even if he/she doesn’t, the attentive instructor can build a certain type of formulated conversation into the lecture to get immediate feedback on the general learning status in the class, and adjust accordingly. The authors have seen from first-hand experience that a positive atmosphere in the classroom leads to the students voicing their questions as often as needed until the learning is achieved. For class conversation formulas that are designed based on neurobiology of learning and that engage, inspire, and enhance retention see [5].

A second practical aspect of Zull’s work also sheds light on the issues of flipped methodology. This aspect has to do with enhancement of learning through evoking of positive emotions. The fact that positive feeling enhances learning has been proven through many studies [3, 4, 6]. The mechanism is as follows. Our sensory data is sent to a brain structure called the thalamus. The signal is then split into two pathways (Fig. 1). The lower pathway, which is activated by negative stimuli, sends the data to amygdala, an almond-shape structure in the brain that is the center for negative emotions. Amygdala, in turn, sends electrical signals to the spine causing reflexes, while at the same time the hypothalamus releases chemicals in the bloodstream causing us to feel angry, embarrassed, or fearful. This pathway is activated not only by explicit negative stimuli, but also by subtler input such as the tone of an instructor or the lighting of the room. An instructor can inadvertently create a caged feeling in the classroom, which is likely to trigger the lower pathway.

**Figure 1:** The two pathways of stimulus processing in human brain

The upper pathway, selected in the presence of positive environmental stimuli, involves the cerebral cortex before sending the data from thalamus to amygdala. This pathway, therefore,
involves the cognitive analysis of an experience that may have initially seemed dangerous but in reality was harmless, or even helpful. Involving cerebral cortex provides more accurate and deeper understanding of the experience. The amygdala responds to positive stimuli by becoming less active, which allows the data to take the upper pathway. The involvement of “thinking” cortex is what enhances learning. This is why creating a positive environment by the instructor boosts learning, because it increases the activity of the upper path and passage of data through the cerebral cortex, another facet that is missing in the out-of-class part of the flip.

The above facts have direct bearing on education. They are the reason why the same technique may work for one instructor and not for another. The techniques themselves are mere tools, but it is the environment stimuli that prompt the brains of our students to choose a certain path, the path leading to more cerebral activity and enhanced learning, or the path to negative reactions and negative behavior. This also has a bearing on classroom behavior. It is the underlying reason why the same group of students may be portrayed by one instructor as dishonest and procrastinating, while another instructor sees only honesty and hard work. Our brains are inherently wired for hard work, since it is associated with dopamine release in the brain. Negative stimuli and fear of judgment can override the positive mode causing learning inhibition and procrastination (for more details and relation to procrastination see [7]).

It is of practical value to also briefly discuss how these learning-enhancing positive emotions can be created in a classroom. Richard Boyatzis, Distinguished University Professor in the Departments of Organizational Behavior, Psychology, and Cognitive Science in Case Western Reserve University [6], led research that has bearing on the "how" of creating those positive stimuli. His studies on leadership were focused on understanding the connection that great leaders form with their audience. His findings showed the effectiveness of a subconscious-level connection very similar to the learning-enhancing instructor-student relationship. His research suggests that the true positive feelings are contagious, through mirroring action of neurons and subconscious perception capability of human brain, meaning that if the instructor indeed "likes" his/her students, the positive stimuli is automatically picked up by the students, who in turn initiate positive signals automatically picked up by the instructor and their peers, leading to a
synergic cycle which, if combined with action (e.g. collaborative problem solving) has the potential of keeping the entire period energized and stimulated. Another part of the "how" is in selection of words to use in class conversations which are an integral part of active learning and collaborative problem solving. Words have powerful emotional associations in human brains, and as such, make great tools for creating positive stimuli. For a good resource on the word choice that inspires participation, see \[5\].

Integration of the above studies, research findings, and practice-tested theories, indeed predicts a missing piece in the out-of-class part of the existing flip methodology, and at the same time, it leads the way toward a redesign to maintain the strength of the methodology and exchange its shortcoming for another strength.

**Our hypothesis.** We mentioned that there has been attempts on making the flipped technique more student-friendly by addressing the issue of learning initiation relying on videos. One such attempt is called the In-Class Flip \[8\]. Research from MOOCs shows that for best engagement, the optimal length of an educational video should be between 6 to 9 minutes \[9\]. This version of the flipped methodology argues that if the lectures are possible to shorten into minutes-long videos, then it should be possible to show those videos in the class, with the instructor present, and devote the remaining time to interactive homework. This approach has its own issues in practice, among them the imperfect fit of regular video-watching time in class, which is still passive lecturing, and also sometimes hard to rationalize as a valid use of class time.

Our hypothesis uses the same initial argument as the In-Class Flip, namely that the lean lectures can very well fit within class time along with interactive problem-solving and collaborative learning. The videos however, are replaced both in-class and before class, with tools from a naturally interactive and brain-stimulating approach called the Visual Thinking Strategy.
**Visual Thinking Strategy (VTS).** Visual Thinking Strategy (VTS) is a relatively simple technique that can be added to most classrooms and most teaching approaches. The technique allows the teacher to deeply embed art into the teaching [10], and in particular into the initiation phase of learning when it matters most to invoke as many brain networks as possible to create a strong basis for the material to be learned. The technique starts the lesson with a carefully prepared image, and simply asking the students what they think is going on. The technique, simple as it is, has proven to be very effective in increasing student engagement, class participation, language skills, writing skills, and visual literacy [10, 11]. Although evidence for the learning-enhancement effect of visual thinking has been available for a long time, its usage in teaching especially in higher education has been largely overlooked. A good review of the research until 1990 is available in [12]. The enhancing connection between reading comprehension and visual imagery was shown for children in third grade throughout their educational career [13]. Similar results were confirmed for fifth graders in a separate study [14], and for learning-disabled students in [15] Another study, [16], showed that first graders learned and retained at a significantly higher rate when imagery was used, and further, the students showed higher level of creativity with usage of imagery [17], a result that can be exploited in higher-education problem-solving. A more recent study [18] reports the effect of using visual thinking software to improve writing skills of students with mild disabilities, and another one [19] provides a practical best practice example on how visual thinking is used to enhance student background knowledge.

Although, these studies were performed at the level of elementary schools, they are readily applicable to higher education classrooms, using thought-provoking images that visually depict the higher concepts to be learned, and as such, VTS has the potential to fit the missing piece of the puzzle in revising the flip.

The result seen in practice in the current study has been impressive. Appendix A contains a detailed narration of one of our VTS-initiated lessons in Mechanics of Materials in the Fall semester of 2016. The narration demonstrates the value of this technique in encouraging
participation in otherwise quiet groups and vastly boosting the pleasure in the discovery part of learning.

**Methods and procedures**

This study consists of two classes of Mechanics of Materials, each taught by one of the authors, one used as the experimental and the other as control. The classes met at the same time during the Fall semester of 2016. The experimental class had 38 students, while the control class consisted of 35 students. Both classes used the same course package containing concepts, derivations, and real-world examples covering a range of basic to advanced skills. The course package was provided by courtesy of Howard and Williams [20]. The textbook was the 10th edition of Mechanics of Materials by R. C. Hibbeler.

The authors selected two course topics for comparison in this study. The two topics were shear and bending moment diagrams and the Mohr’s circle. In teaching those two topics, one instructor followed the traditional techniques (see the subsection on Methods used in the control class,) while the other instructor implemented the Revised Flip with VTS.

For assessment, the students in both classes received the same questions on the two selected topics, and their performance was measured according to a common rubric provided in Appendix B.

**Methods used in the experimental class.** The experimental class started by an informal discussion with the students on their opinion about the flipped classroom. The students, majority of whom had previous experience with flipped classrooms, voiced their feedback, which was in line with the general feedback available on flipped strategy, i.e. the students disfavored premade videos but favored the interactive problem solving. The class therefore replaced the videos with VTS tools, on the two topics that were to be compared with the control class.
On the topic of shear and bending moment diagrams, the pre-class VTS tool consisted of a static beam problem solved concisely and visually. The solution contained the calculation of shear and bending moments. The problem was a statics problem, which contained no new material, but evoked and refreshed their prerequisite knowledge in the areas of interest. This solved problem was a short visually appealing PDF document (see Appendix C). The second part of this document illustrated the sign convention that was to be used in mechanics of materials for shear and moment diagrams. The sign convention was also presented with images and very few words. A short online quiz was used as formative assessment, enabling the students to check their understanding of the concepts of this exercise (see Appendix C).

Student reaction to this assignment was quite favorable. The class indeed started with an active discussion and critiquing of the details of the convention, a discussion that was initiated by the students themselves. The instructor then went on to introduce the diagrams, in a short lecture, and the rest of the time was devoted to collaborative problem solving on the topic.

**Collaborative problem-solving.** The problem-solving part to the flipped methodology was performed in the experimental class with collaborative problem-solving. For this purpose, the instructor provides a problem, which is usually selected in the intermediate range of difficulty. The problem is designed to allow the students to test their learning of the basics, but also to build upon the introductory concepts with collaborative discussion and critical thinking. The students start by reading and setting-up the problem individually, taking 5-7 minutes. They then work in small groups loosely formed by their own preferences. Usually there is walking and consulting among the students and peer teaching happens quite frequently. The instructor periodically joins different groups and helps answer questions, preferably by asking other questions, which lead the students to more critical thinking, forming theories, and testing theories on their own. In the end, the students write a solution report and turn it in for an individual grade. These collaborations are also used as a means of individualized instruction, as the instructor reads them and gives individual feedback to the students on both the accuracy and communication of the solutions.
On the topic of Mohr’s circle, the pre-class assignment consisted of an unexplained animation of a stress element rotating from its original state to the principal axes state. The animation showed the normal and shear stresses changing as the element rotated [21].

It is worthwhile to mention that for the first assignment the quiz was used for the dual purpose of students testing themselves and also a grade incentive to ensure participation. On the next pre-class assignment, however, the animation had no quiz attached, but this did not reduce participation by any means. The students had indeed watched the animation and were excited to discuss it.

Again, the class started with a lively discussion initiated by the students themselves and circling around what they thought this animation represented. The animation provided discussion material and visual context for the Mohr’s circle derivations. The instructor referred back to it in multiple occasions and the student comprehension of the material was enhanced. The next part of the class time was devoted to collaborative problem solving. The students were asked to bring a compass and drawing tools to draw Mohr’s circles, discuss it, and make it their own. This turned to a learning experience for the instructor too, when she realized the compass tool was indeed obsolete, as many of the students had drawing apps to take its place. Mutual learning always creates deep synergy and an uplifting atmosphere.

**Methods used in the control class.** The teaching approach used in the control class is the somewhat standard in-class lecture, out-of-class homework format, with a few notable exceptions. The in-class lectures heavily relied on a course package of material of which the students would print out in advance of class (generally chapters at a time) and use during class to take notes. The course package follows the textbook, streamlines introductory content, provides initial diagrams for derivations, leaves plenty of room for filling in derivations, key concepts and definitions, and provides example problem statements for working in-class. The key advantage
of the course pack is that it compresses the lecture time and allows for more in-class problem solving time. Unlike the experimental section, students are not explicitly encouraged to move around or collaborate, although they often will work with their immediate neighbor. Also the in-class exercises are not taken up for grading or feedback. However, the exercises are a feedback vehicle, as the instructor will randomly call on students by name, asking for key concepts, solution approaches, next steps, and intermediate and final answers. Often, student misconceptions can be identified and corrected during these in-class problem solving exercises.

Shear and bending moment diagrams were introduced, developed, and reinforced over the course of three lectures. Initially, the concept of beam loading and internal shear and bending moments are related to previously studied axial and torsional loading. The general equations for the shear and moment as a function of beam length are then derived for a simply supported beam with a concentrated load at mid-span. The students are encouraged to develop their own free body diagrams and sum the forces and moments to derive the shear and moment equations. Once derived, the equations are plotted as a function of beam length. This exercise is repeated for a simply supported beam with a uniformly distributed load and a cantilever beam with a concentrated load at the free end. At the end of these exercises, the class reviews all of the shear and moment diagrams and are asked for their observations regarding relationships between the shear and the moment. Most often a few students readily can identify several of the relationships. With these concepts freshly revealed, the relationships are derived (guided by the course package), using various loads on differential elements. At the end of the derivations, the rules are summarized. The material presented here normally takes 2 one-hour lectures to cover. During a third lecture, three problems of increasing difficulty are work in-class by the students using the approach described above. It should be noted that the expectation of neatness and accuracy (plotting the diagrams to scale), including the use of a straight edge, is set and reinforced. The lesson concludes with a homework assignment of four shear and bending moment problems.

The presentation of Mohr’s circle follows a similar format, starting with the review of stresses on a scarf joint of a previously studied axially loaded member. The students are coaxed into
recognizing that the shear and normal stress will depend on the angle of the scarf, leading to a guided derivation of the planar stress transformation equations. Once the equations are derived, the class is asked for which stress is most important and why, which leads to the definition of principal stresses. The stress transformation equations are then plotted for a 360° rotation of the axes, leading to a discussion of observations of the relationships between the three stresses. Finally, Mohr’s circle is introduced as a graphical method of finding the principal stresses and maximum in-plane shear stress in lieu of the mathematical rigor of the stress transformation equations. The instruction of Mohr’s circle consisted of providing the students with a step-by-step approach and then guiding them through two in-class exercises followed by a homework assignment of three problems. Again, it should be noted that the expectation of neatness and accuracy, including the use of a straight edge and compass, is set and reinforced.

**Results and discussion**

A quantitative measure was provided through the final exam. The students in both experimental and control classes received the same questions on the two topics, shear and bending moment diagrams and the Mohr’s circle, as selected for comparison. The solutions were evaluated based on the same rubric, provided in Appendix B. The details of the results follow in Figures 2 and 3.

![Figure 2: Student performance in final exam for the compared topic Problem 1: Shear and moment diagrams](image-url)
While a more conclusive discussion of the data requires the study to include more sections and different instructors to account for factors such as the group make-up, and instructor experience, the following observations can still be made from the data:

1. The number of students who made conceptual mistakes was low in the experimental class. This is shown by the percentage of students who performed 4/5 and above. This number for problem 1 was 63% for the experimental class compared with 54% for the control class. When comparing the percentage of class who made above 3/5 for problem 2, the number is 92% for the experimental class as opposed to 74% in the control class. A similar, but less pronounced, trend is visible when making the comparison for percentage above 4/5 for problem 2 (66% in experimental versus 60% in control) and percentage above 3/5 for problem 1 (89.4% versus 85.7% in control).

This observation supports the predicted effect of enhanced engagement and student participation, confirming in practice the neurobiological advantages of both VTS and collaborative problem-solving in helping to reach and engage a larger number of students.
2. The control class shows a very high proficiency in accuracy. A discussion of control class’s techniques revealed the instructor’s specifically targeting of accuracy with tools mentioned in the section on the control class methods. These techniques could be studied on their own, and would make a great addition to the experimental class, which is what the instructors implemented in a follow-up semester.

**The follow-up semester.** In a follow-up semester, the same study was repeated with the same instructors covering parallel sections, and performing a similar comparison with common final exam questions on the same topics. The only difference was that the experimental class added the emphasis on accuracy, learned and adopted from the results of the control class. This emphasis increased the percentage above 4/5 of problems 1 to 75%, a ten percent increase compared with the previous semester, confirming the efficacy of the control class’s specific emphasis on accuracy. The same number for problem 2 reached 92%, again an increase in achieving accuracy.

**Student feedback on the experimental class**

In our hypothesis, we proposed that the alternative to flipped classroom would have the effective engagement of the flip without the student dislike of it. The effective engagement and reaching more students was supported by the above results. Moreover, a narration of one of our sessions is provided in the Appendix A, which also demonstrates the active engagement. The final part of the hypothesis is supported by the students’ anonymous evaluation of the course at the end of the semester for the experimental class. This was the tool that confirmed the successful resolution of the unpopularity of the flip. A few of the related rating and comments are provided in Table 1.
Table 1: Student evaluation data of the experimental class

<table>
<thead>
<tr>
<th>Evaluation area</th>
<th>Student rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Encouragement of critical thinking</td>
<td>5.6 /6</td>
</tr>
<tr>
<td>Creation of a conducive atmosphere for learning</td>
<td>5.5 /6</td>
</tr>
<tr>
<td>Clarity of the material</td>
<td>5.2 /6</td>
</tr>
<tr>
<td>Overall learning</td>
<td>5.4 /6</td>
</tr>
</tbody>
</table>

**Excerpts from anonymous student comments.** “The collaborative questions forced me to think, and then helping others helped me to fully understand the material. Teaching others helps to concrete things to me…”

“… class structure is ideal for the hard engineering classes with collaborative problems we work on with other students in the class and the problems worked in class.”

**Conclusions**

A variation to in-class flip methodology was designed, by replacing pre-class and in-class videos of flipped lessons with Visual Thinking Strategy tools. The technique was tested and the result was enhancement of student engagement and participation, facilitation of learning, and favorable student feedback.

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Appendix A – A narrative of one of our VTS sessions

The experimental call of Mechanics of Materials on method of superposition in deflection of beams – Fall 2016. Student names are changed for privacy.

In the beginning of the class, the instructor displayed the image shown in Fig. 4 on the screen for students to examine. No explanation was given.

![Figure 4: VTS tool for teaching method of superposition in calculating beam deflections](image.png)

Instructor: What do you think is going on here?

John: We have beams and forces.

Dustin: It’s adding, (looking at the instructor) Is this saying that we can add forces?

Instructor: That’s a good thought. Dustin is asking what is it that we are adding here?

David and Scott: Bending!

Luke: Deflection! It is deflection!

Instructor: It is deflection!

Dustin (skeptically): Are you saying we can add deflections? Just make a table instead of all those differential equations, boundary conditions?

Instructor: (nodding excitedly) Isn’t it great?

Josh (even more skeptically): Why would that be valid?

John: Yes, why?

Instructor: Why do you think? What was the formula for deflection? What was it related to?
A moment of silence and murmuring in class.

Josh looks up and recites the formula for deflection and its relations to bending moments.

Leah: Moments! It’s linearity. It is linear, so we can add.

Instructor confirms. The students now know everything about the topic, discovered and reasoned by themselves, attached to a telling image that makes for a powerful memory. The rest of the class focuses on collaborative problem solving using superposition tables, exploring the tricks and shortcuts, making it their own.

Appendix B – Common grading rubric

Each graded problem was evaluated on a 0 to 5 integer point scale with the following descriptions. Complete show of work was required for partial credit.

5: Accurate formulation and error-free results, neatly communicated

4: Accurate formulation, but inaccurate calculations (algebra mistakes)

3: Minor conceptual mistakes in formulation

2: Major conceptual mistakes in formulation

1: Minimal attempt at solution

0: No solution

Appendix C – Sample pre-class preparation assignment

The following visually-rich reading material was assigned alongside the quiz, due before the class on shear and bending moment diagrams.
PRE-CLASS SUMMARY READING

SHEAR AND BENDING MOMENT DIAGRAMS

Instructions. Read the following summary and take the related quiz on Canvas.

Due date. Monday 9/26/16 11:59 p.m.

Beams analysis. We have so far analyzed structures in axial loading, and torsional loading, but not transverse loading. Now we study loading perpendicular to the axis of the structural members.

As an example, a railway bridge and the load of the train on it can be modeled as

To analyze the stresses on the bridge we will need to section the beam and look at internal forces and moments.
**Sign convention.** When looking at a section, we need a convention for directions, and this is the common practice:

Consider the section at x, shown in the image. (In general there will be loading on the beam.)

<table>
<thead>
<tr>
<th>Left segment</th>
<th>Right segment</th>
</tr>
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<tbody>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
</tr>
<tr>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
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</tbody>
</table>

Positive $M(x)$ (shown in image on both sections) subjects the upper part of the beam to compression and the lower part of it to tension.

Positive $v(x)$ (shown in image on both sections) turns the beam cw.
Positive $P(x)$ (shown in image on both sections) is tension.

Note that the sign convention above applies to pairs of sections. For each section, we use the usual conventions to sum up the forces and moments as shown in the example that follows.
**Example.** The beam in the figure is subject to a distributed load of 7500 N/m, as shown. Find the shear and bending moment at point A.

**Solution:**

First consider the overall free-body diagram:

\[ \sum M_O = 0 \Rightarrow 375 \, By - (250) \cdot (750) = 0 \]
\[ \Rightarrow By = 500 \, N \]

Then make a section at A, and take the right segment:

\[ \sum F_x = 0 \Rightarrow P_A = 0 \]
\[ \sum F_y = 0 \Rightarrow V_A + By - 375 = 0 \]
\[ \Rightarrow V_A = -125 \, N \]
\[ \sum M_A = 0 \Rightarrow \\ B_y \times (375 \, mm - 250 \, mm) - 375 \, N \times \frac{50}{2} \, mm = M_A = 0 \Rightarrow M_A = 53.125 \, N.m \]

**Critical thinking.** If we need P, V, and M at any x, what method do you suggest for finding them? What would those diagrams look like?
Pre-class reading quiz. The following problem was used as a quiz. Multiple choices of shear and bending moment diagrams were provided for students to choose from. This task did not require them to generate the diagrams, which they had not yet been taught about, but it did encourage them to connect the dots and find ways to test the multiple choices and eliminate the wrong ones, thus forming a good foundation for the actual instruction that would come in class.

Quiz question. Consider the beam below. Which pair correctly represents the shear V(x) and bending moment M(x) diagrams.

Two attempts were allowed to facilitate learning through usage of immediate feedback.

References


